reverse. Eventually, after a short run in the diffuser, the boundary layer on what was the cathode wall exhibits a higher shape factor and lower skin friction than that on the opposing wall. This is a reaction to the sudden relief of the transverse Lorentz force. This force was formerly directed toward the wall in the cathode boundary layer and away from the wall in the anode boundary layer. When the channel unloads, these forces are no longer present and the boundary layers are suddenly out of force balance as well as suddenly acted upon by an adverse pressure gradient (rather than the favorable gradient present in the active channel). These calculations indicate that for diffusers of practical length, the asymmetry will persist with the boundary layer on what was the cathode wall being considerably more susceptible to separation. Note also in Fig. 3 the return to conventional behavior of the sidewall skin friction as the axial Lorentz force is relieved. The somewhat more abruptly changing behavior in the distributions of the boundary-layer parameters at approximately 9.25 m is again a consequence of the geometry. At this point the electrode walls become parallel while the divergence half angle on the sidewalls is reduced from 2.0 to 1.5 deg. In addition, the electrode wall surface temperature drops significantly in this region due to a change in surface composition, hence the abrupt rise in the heat flux shown in Fig. 4.

References

[1] Demetriades, S. T., "Momentum Transfer to Plasmas by Lorentz Forces," *Physico-Chemical Diagnostics of Plasmas,* Northwestern Univ. Press, Evanston, Ill., 1964, pp. 297-328.

[2] Norman, W., and Siler, L. G., "Experiments on a Magnetohydrodynamic Nozzle Accelerator in a Shock Tunnel," Arnold Engineering Development Center Tech. Rept. AEDC-TR-63-53, March 1969.

[3] Doss, E. D., Argyropoulos, G. S. and Demetriades, S. T., "Two-Dimensional Flow Inside MHD Ducts with Transverse Asymmetries," *AIAA Journal,* Vol. 13, May 1975, pp. 545-546.

[4] Argyropoulos, G. S., Demetriades, S. T., and Kendig, A. P., "Influence of Nonuniformities, Thermal Diffusion, Boundary Layers and Finite Reaction Rates in Nonequilibrium J×B Devices," AIAA Paper 67-715, Sept. 1967.

[5] Demetriades, S. T. and Argyropoulos, G. S., "Ohm's Law in Multicomponent Nonisothermal Plasmas with Temperature and Pressure Gradients," *Physics of Fluids,* Vol. 9, No. 11, Nov. 1966.

[6] Argyropoulos, G. S., Demetriades, S. T., and Lakner, K., "Compressible Turbulent Magnetohydrodynamic Boundary Layers," *Physics of Fluids,* Vol. 11, Dec. 1968, pp. 2559-2566.

[7] Demetriades, S. T., Argyropoulos, G. S., and Maxwell, C. D., "Progress in Analytical Modeling of MHD Power Generators," *Proceedings of 12th Symposium on Engineering Aspects of Magnetohydrodynamics,* Argonne National Lab., March 1972, pp. I.5.1-I.5.13.

[8] Demetriades, S. T., et al., "STD/MHD Code Descriptions for Computer Programs Utilized under Contract AC-01-79-ET15501," STD Research Corp. Rept. No. STDR-79-3, Jan. 1979.

[9] Demetriades, S. T., et al., "Numerical Solutions of the Coupled Three-Dimensional Magnetohydrodynamic Channel Problem," STD Research Corporation Final Rept. No. STD-75-001 prepared for the National Science Foundation under Contract NSF-C-727, Jan. 1975.

[10] Van Driest, E. R., "On Turbulent Flow Near a Wall," *Journal of the Aerospace Sciences,* Vol. 23, 1956, p. 1007.

[11] Markham, D. M., Maxwell, C. D., and Oliver, D. A., "A Numerical Solution to the Unsteady, Quasi-Three-Dimensional, Turbulent Heat Transfer Problem in an MHD Channel," ASME Paper 77-HT-90, Salt Lake City, Aug. 1977.

[12] Demetriades, S. T., Argyropoulos, G. S., Curry, B. P., and Patel, N. J., "Numerical Solutions of the Coupled Three-Dimensional Magnetohydrodynamic Channel Problem," STD Research Corp. Final Rept. No. STD-75-001 prepared for the National Science Foundation under Contract NSF-C-727, Jan. 1965.

[13] *Preliminary Proceedings of AFOSR-IFP-Stanford 1968 Conference on Turbulent Boundary Layers, Vol. II: Compiled Data,* Stanford, Calif., 1968.

[14] Maxwell, C. D., Demetriades, S. T., Oliver, D. A., Vetter, A. A., Swean, T. F., "Scale-Up of Advanced MHD Generators," AIAA Paper 80-0179, Pasadena, Calif., Jan. 1980.

AIAA 81-4109

Mixing Length in Low Reynolds Number Turbulent Boundary Layers

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Nomenclature

G = Clauser shape parameter, $\sqrt{(2/C_f)}\ (1-1/H)$
H = shape parameter, δ^*/θ
K = pressure gradient parameter, $(\nu/U_e^2 \mathrm{d}U_e/\mathrm{d}x)$
l = mixing length
M = Mach number
u_τ = friction velocity, τ_w/ρ_w
β = pressure gradient parameter, $(\delta^*/\tau_w \mathrm{d}p/\mathrm{d}x)$
Δ_p = pressure gradient parameter, $(\nu/\rho u_\tau^3 \mathrm{d}p/\mathrm{d}x)$
δ = velocity boundary-layer thickness where $U/U_e = 0.995$
δ^+ = Reynolds number, $u_\tau \delta/\nu_w$
δ^* = boundary-layer displacement thickness
τ_w = wall shear stress
ν = viscosity
ρ = density

Subscripts

m = maximum value
0.5 = evaluated at $y/\delta = 0.5$
e = edge
w = wall

Introduction

BUSHNELL et al.[1] analyzed existing data to show that, at low values of the Reynolds number δ^+, the mixing-length parameter $(l/\delta)_m$ was smaller at large distances downstream of transition than at small distances, for given δ^+. The data at small distances came from measurements on flat plates and other isolated bodies, whereas the measurements at large distances were made on the nozzle walls of supersonic wind tunnels. Comparison of Table 1 and Fig. 2 of Ref. 1 shows that nearly all the measurements which show a lower than usual mixing-length parameter were taken at Mach numbers above 5. Existing low-speed data (e.g., Refs. 2 and 3) indicate that the mixing-length parameter increases with decreasing Reynolds number on flat plates, so that the question of whether the reduction found by Bushnell et al. occurs only at high Mach numbers or whether it is a universal feature of low-Reynolds-number turbulent boundary layers remained open. This Note presents the results of an experimental and computational investigation of this question.[4]

Low-Speed Experiment

Experimental Arrangement

To produce a low Reynolds number δ^+ far downstream of transition we set up, in the Imperial College 0.762×0.127 m (30×5 in.) boundary-layer tunnel,[5] a flow with a favorable pressure gradient sufficiently strong to keep the momentum-thickness Reynolds number at or below 1000 for a distance of about 50 boundary-layer thicknesses. Over this distance, the momentum thickness Reynolds number on a flat plate would approximately double. The initially laminar boundary layer was tripped by a spanwise wire sized to ensure rapid recovery from transition of the inner layer,[6] as demonstrated by the

Received Aug. 6, 1980; revision received Dec. 9, 1980.

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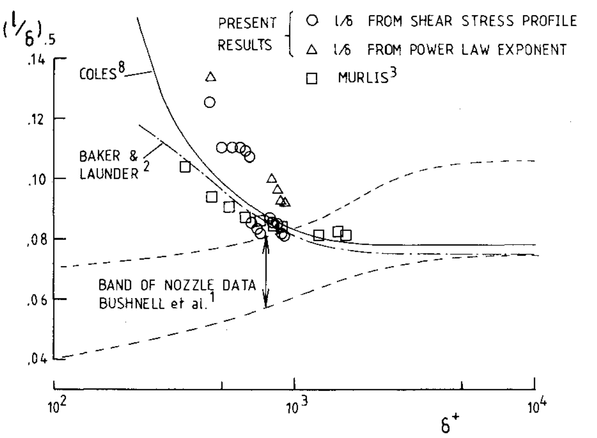

Fig. 1 Mixing lengths in outer region of low-speed accelerated turbulent boundary layer.

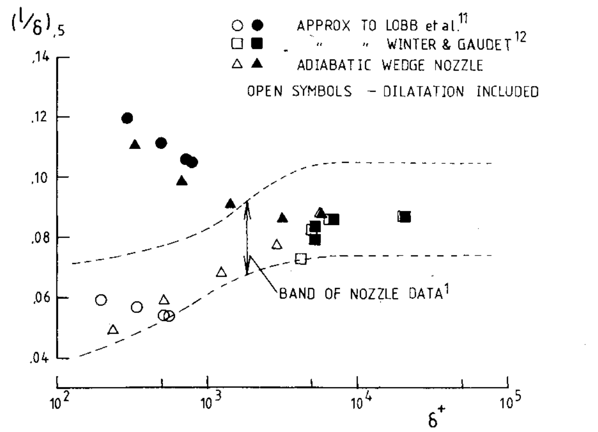

Fig. 2 Calculated mixing lengths in outer region of supersonic nozzle flow turbulent boundary layers with and without allowance for dilatation.

development of a well-defined logarithmic profile in the 20 layer thicknesses upstream of the acceleration region. Over the main part of the acceleration region the pressure-gradient parameter K was nearly constant at 0.85×10^{-6}, the more meaningful inner-layer pressure-gradient parameter Δ_p was about -0.0075, and the pressure-gradient parameter β representing the ratio of the pressure-gradient term in the momentum integral equation to the shear-stress term was about -0.5 (close to its value in the experiment of Herring and Norbury,[7] that is generally accepted as a truly turbulent boundary layer without trace of relaminarization). Downstream of the region of acceleration, the flow developed in zero pressure gradient up to a Reynolds number based on momentum thickness of 2500; the Clauser shape parameter G, which reached a minimum value of 5.5 in the acceleration region, had nearly reached the asymptotic value of about 7.0 at the end of the test section, and the Coles' wake parameter, though still below its high-Reynolds-number value, agreed with the consensus of experimental results plotted by Coles.[8]

Data Analysis Techniques and Results

Mixing lengths evaluated from the present data are presented in Fig. 1. The variations of mixing length with δ^+ found by Baker and Launder[2] and Murlis[3] are also shown, together with the variation deduced from the profile family suggested by Coles. An indication of the extent of the nozzle data from Fig. 2 of Ref. 1 is given. Within the errors of determination the present results are consistently above the high-Reynolds-number value. In the acceleration region the mixing-length parameter falls from its initially high value, but shows no excursion to values lower than the established data.

The evaluation of mixing length in Ref. 1 was based on the use of power-law fits to velocity profiles and assumed a linear shear-stress distribution through the boundary layer. The present results have been analyzed both with this approximate technique (for profiles where the linear shear-stress approximation is valid) and with the real velocity and shear-stress profiles. The latter were evaluated from the momentum equation, with an allowance for three-dimensionality deduced from the imbalance in the momentum integral equation. No direct turbulence measurements were made.

Discussion

The acceleration chosen for the present experiment may be considered mild, but in fact boundary layers with stronger acceleration parameters were found to exhibit significant deviations from the logarithmic law of the wall. The presence of this deviation was demonstrated both by comparing skin friction measured by Preston tubes (which rely on the presence of a logarithmic region) and small sublayer fences (which rely only on the universality of the viscous sublayer) and by inspection of detailed profile measurements.

The present experiment constitutes a careful search for possible decreases in mixing-length parameter in low-speed flows without relaminarization. The absence of any decrease in mixing length therefore suggests that the effect found by Bushnell et al. in their analysis of measurements on supersonic wind tunnel nozzle walls is in fact confined to high Mach numbers. Bushnell et al. conclude that the anomalous low-Reynolds-number effects found on nozzle walls are different from the anomalous effect of wall temperature and pressure-gradient history well known to occur on tunnel walls. Since the present results imply that distance from transition as such is not an important parameter at low speeds, it is unlikely to be responsible for the high-speed effect on mixing length. The fact that the mixing-length parameter in flat-plate flows exhibits much the same behavior (a rise with decreasing Reynolds number) at both low speeds and high speeds shows that the decrease in mixing length found on wind-tunnel nozzle walls is not a pure Mach number effect.

Compressible Flow Calculations

The most likely explanation of the results of Ref. 1 is a persistence of upstream history effects on wind tunnel nozzle walls. In particular, it is known that large negative pressure gradients in compressible flow decrease turbulence intensity,[10] the decrease in density (bulk dilatation) having a considerable effect on turbulence structure. In order to check this a series of calculations has been made by the method of Ref. 9 for simulated supersonic wind tunnel nozzle pressure gradients up to a Mach number of 5, both with and without the allowance for dilatation effects recommended in Ref. 10. The test cases include close approximations to the low and high Reynolds number flows of Lobb et al.[11] and Winter and Gaudet,[12] respectively; both flows were used in Fig. 2 of Ref. 1. The intermediate-Reynolds-number range was covered by a series of hypothetical wedge nozzle flows varying from a Mach 1 zero-pressure-gradient flow to a Mach 5 flow similar to the Lobb et al. cases. The calculated mixing lengths shown in Fig. 2 indicate that the low values observed on nozzle wall flows can be explained by the effects of bulk dilatation of the flow that results from the acceleration necessary to produce low values of δ^+ at large distances from transition.

Discussion

Recognition of the importance of bulk dilatation in the turbulent kinetic energy equation implies the inclusion of normal-stress and pressure-gradient terms previously assumed insignificant, and the factoring of the production/dissipation terms to represent influence of the small extra rate of strain. Implementation in the current model is correct to first order only and gives rise to a single extra term incorporating both effects, their relative contributions being approximately equal. Dilatation effects scale roughly with M_e^2; at higher Mach numbers the extra term can be of the same order as, say, the production term. In such cases the rate of strain clearly is not small and calls the accuracy of the first-order model into question.

The choice of Reynolds number in compressible flow is critical, since differences amounting to a factor of 20 can arise at a Mach number of 5 depending on whether wall or freestream values of density and viscosity are specified. The evidence[3,13] suggests that low-Reynolds-number influence is located in the viscous superlayer, implying the use of freestream values. In these terms nozzle flows do not exhibit low Reynolds numbers. Mixing lengths predicted without dilatation and low-Reynolds-number corrections (Fig. 2) are high primarily because of the favorable pressure gradients. The effect of a low-Reynolds-number correction is to increase the results of Fig. 2 by a small factor whose magnitude depends upon the definition of Reynolds number employed.

Conclusions

The evidence of Coles' profiles for zero pressure gradient suggests that even in the absence of transitional effects a rise in mixing length will occur at low Reynolds numbers. The persistence of transitional effects (e.g., overshoot of the power-law exponent, reduced wake strength) results in a further increase. The effect of acceleration as such is to retard the growth of the boundary layer, and hence the wake component, and if there are no further complicating effects the mixing length remains high. We propose that the low values found by Bushnell et al. on supersonic nozzle walls are a result of a further complicating effect, the modification of turbulence structure by bulk dilatation in highly-accelerated compressible turbulent boundary layers.

Acknowledgment

We are grateful to Mr. Dennis Bushnell for comments on an early draft of this Note.

References

[1] Bushnell, D. M., Cary, A. J. Jr., and Holley, B. B., "Mixing Length in Low Reynolds Number Compressible Turbulent Boundary Layers," *AIAA Journal*, Vol. 13, Aug. 1975, pp. 1119-1121.

[2] Baker, R. J., and Launder, B. E., "The Turbulent Boundary Layer with Foreign Gas Injection. –1. Measurements in Zero Pressure Gradient," *International Journal of Heat and Mass Transfer*, Vol. 17, 1974, pp. 275-291.

[3] Murlis, J., "The Structure of a Turbulent Boundary Layer at Low Reynolds Number," Ph.D. Thesis, Dept. of Aeronautics, Imperial College, Univ. of London, 1975.

[4] Inman, P. N., "Transitional and Low Reynolds Number Turbulent Boundary Layers," Ph.D. Thesis (in preparation), Dept. of Aeronautics, Imperial College, Univ. of London, 1981.

[5] Bradshaw, P., "Two More Wind Tunnels Driven by Aerofoil-Type Centrifugal Blowers," Dept. of Aeronautics, Imperial College, Univ. of London, Aero. Rept. 72-10, Feb. 1972.

[6] Graham, J.M.R., "The Development of the Turbulent Boundary Layer Behind a Transition Strip," Aeronautical Research Council, London, 31,492 F.M. 4090, Oct. 1969.

[7] Herring, H. J. and Norbury, J. F., "Some Experiments on Equilibrium Turbulent Boundary Layers in Favourable Pressure Gradients," *Journal of Fluid Mechanics*, Vol. 27, 1967, pp. 541-549.

[8] Coles, D., "The Turbulent Boundary Layer in a Compressible Fluid, Appendix A, A Manual of Experimental Boundary Layer Practice for Low-Speed Flow," Rand Corp., Santa Monica, Calif., R-403-PR, Sept. 1962.

[9] Bradshaw, P., Mizner, G. A., and Unsworth, K., "Calculation of Compressible Turbulent Boundary Layers on Straight-Tapered Swept Wings," *AIAA Journal*, Vol. 14, March 1976, pp. 399-400.

[10] Bradshaw, P., "The Effect of Mean Compression or Dilatation on the Turbulence Structure of Supersonic Boundary Layers," *Journal of Fluid Mechanics*, Vol. 63, 1974, pp. 449-464.

[11] Lobb, R. K., Winkler, E. M., and Persh, J., "NOL Hypersonic Tunnel No. 4 Results VII: Experimental Investigation of Turbulent Boundary Layers in Hypersonic Flow," NOL NAVORD Rept. 3880, 1955.

[12] Winter, K. G. and Gaudet, L., "Turbulent-Boundary Layers at High Reynolds Numbers at Mach Numbers Between 0.2 and 2.8," Aeronautical Research Council, London, R&M 3712, 1973.

[13] Falco, R. E., "Coherent Motions in the Outer Region of Turbulent Boundary Layers," *The Physics of Fluids*, Vol. 20, Oct. 1977, pp. S124-S132.

AIAA 81-4110

Scaling of Interactions of Cylinders with Supersonic Turbulent Boundary Layers

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Introduction

THE upstream extent of the disturbed flowfield induced by a cylindrical protuberance (or hemi-cylindrically blunted fin) in a supersonic turbulent boundary layer has been studied experimentally by many investigators.[1-13] In the past, protuberances have generally been classified as small or large[2] "on the basis of their height, h, relative to the 99% velocity boundary layer thickness δ." A protuberance is generally considered to be large if it produces the "asymptotic result," a condition occurring when further increases in h do not increase the extent of the disturbed flowfield. It has been noted[2] that "the asymptotic result is obtained rather quickly for $h>\delta$." In contrast, in a recent study[3] at a freestream Mach number M of 5.3, cylindrical protuberances of different diameters D were classified as being small if $h<D$ and large if $h>2D$. However, no physical explanation was offered for the latter result, nor mention made as to whether it should be interpreted as a general conclusion.

This apparent contradiction raises questions of which geometric and flow parameters are physically significant, and under what conditions is the asymptotic result obtained. These are the questions addressed by this Note. In addition, a simple approach is outlined for estimating if a cylindrical protuberance of given dimensions will generate the asymptotic result. In the following discussion, the incoming boundary layer is assumed turbulent. A laminar boundary layer does not change the basic approach, but does modify the quantitative result, as will be mentioned later.

Discussion

Physically, the asymptotic result occurs when the root shock structure is isolated from the free end by a two-dimensional central region in which the leading edge surface pressure Pw is equal to the freestream pitot value Pt_2. This condition is sketched in Fig. 1a. Typically, it occurs when h/htp is about 2-3, where htp is the asymptotic triple point height. Increasing h further only increases the extent of the central region, with no effect on the root region. Decreasing h, as shown in Fig. 1b, eliminates the central region, causing an interaction between the root and free-end flowfields. This, in turn, results in a reduction of asymptotic centerline upstream influence Lu.

Received Sept. 16, 1980; revision received Feb. 2, 1981.

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